

# Table of Contents

---

---

---

<b>Lesson One</b>	Whole Numbers.....	3
<b>Lesson Two</b>	Common Fractions.....	21
<b>Lesson Three</b>	Decimal Fractions.....	39
<b>Lesson Four</b>	Ratios and Proportion.....	55
<b>Lesson Five</b>	Powers and Roots.....	71
<b>Lesson Six</b>	Calculators.....	87
<b>Lesson Seven</b>	Geometry.....	105
<b>Lesson Eight</b>	Algebra.....	123
<b>Lesson Nine</b>	Using Formulas.....	141
<b>Lesson Ten</b>	Trigonometry.....	157

© Copyright 1982, 2001, 2009 by TPC Training Systems, a division of Telemedia, Inc.

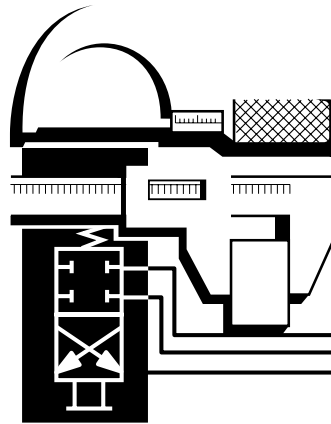
All rights reserved, including those of translation.

Printed and videotaped courseware are subject to the copyright laws of the United States. You are not authorized to make any copies of this material. If you do, then you are subject to the penalties provided under the copyright law, which include statutory damages up to \$50,000 for each infringement of copyrighted material, and also recovery of reasonable attorneys' fees. Further, you could be subject to criminal prosecution pursuant to 18 U.S.C. § 2319.

***MATHEMATICS IN THE PLANT***

***Lesson One***

***Whole  
Numbers***



***TPC Training Systems***

10301

**Lesson****Whole Numbers****TOPICS**

**Numbers and Numerals**  
**Decimal Systems**  
**Positive and Negative Numbers**  
**Addition**  
**Carrying**

**Shortcuts in Addition**  
**Multiplication**  
**Subtraction**  
**Borrowing**  
**Division**

**OBJECTIVES**

**After studying this lesson, you should be able to...**

- Describe the difference between a number and a numeral.
- Demonstrate how to add three four-digit numbers, with carrying.
- Demonstrate how to subtract two four-digit numbers, with borrowing.
- Demonstrate how to multiply a four-digit number by a two-digit number.
- Demonstrate how to divide a four-digit number by a two-digit number.

**KEY TECHNICAL TERMS**

**Number** 1.01 an idea

**Numeral** 1.01 symbol for a number

**Digit** 1.05 symbol for a whole number from zero to nine

**Decimal system** 1.08 number system based on ten

**Place value** 1.15 value indicated by a digit's place in a numeral

Many people, including maintenance specialists, feel unsure of their ability to solve problems with numbers. But using numbers to solve problems in the shop is an important skill to master if you are to be a good maintenance worker. The purpose of this course is to show you how to use math in everyday maintenance work.

This lesson explains the four basic math operations—addition, multiplication, subtraction, and division. You probably remember most of these skills from your early days in school. If you do, you can use this lesson simply to refresh your memory. It may also help you understand more about the reasons for certain things you learned in school. If you have forgotten part of your math skills, this lesson will help you relearn those parts.

Throughout this lesson and in the lessons that follow, you will find sets of practice problems. Take the time to solve these problems and to write your answers in the spaces provided. Then check your answers on the last page of the lesson.

Do not skip the problems. If you are to be a professional maintenance specialist, you should always be willing to prove to yourself that you have all the necessary skills.

## Numbers and Numerals

1.01 A *number* is an idea that someone has in mind. A *numeral* is a symbol used to express a number.

1.02 When you write “3 machines” or “5 minutes” or “20 feet,” you are using numerals to express ideas. The numerals stand for the number of machines, the number of minutes, and the number of feet.

1.03 Figure 1-1 shows three sets of screws. How many screws are there in Set 1 and 2? Write your answer in this space: \_\_\_\_\_  
How many in Set 3? \_\_\_\_\_

1.04 To find out how many screws are in each set, you counted them. Counting is the process of naming whole numbers in their proper order: 1, 2, 3, 4, 5, 6, etc.

1.05 Primitive people used their ten fingers to express numbers visually. To express “two spears,” a hunter could hold up two fingers. He had a problem if he wanted to express the number 50 or 100, because he did not have enough fingers. Fifty, to the primitive man, may have meant “all my fingers, five times.” This idea may have been the start of the number system we use today. In fact, the ten symbols we use to express whole numbers from zero through nine are called *digits*. The word digit comes from the Latin word *digitus*, meaning finger or toe.

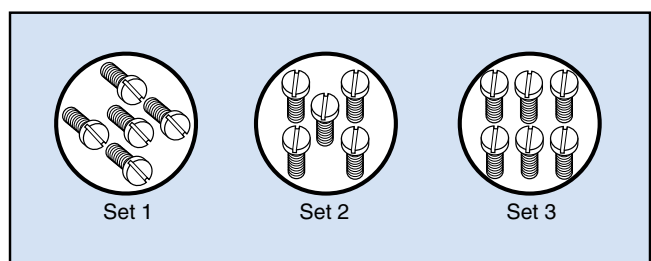
## Decimal System

1.06 Different civilizations throughout history used different numerals to keep records and accounts. You may be familiar with Roman numerals. These numerals were once used for writing all numbers. Today, most of the world uses the *Hindu-Arabic system*.

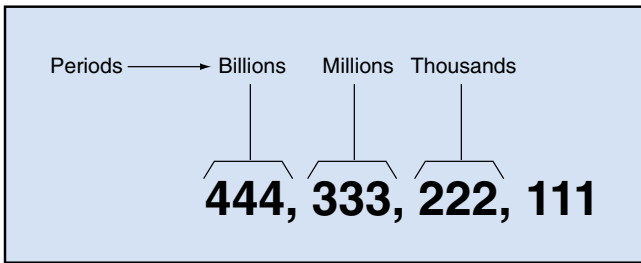
1.07 Sometime before 700 A.D., Hindu mathematicians began using a *place value* system and a symbol for zero. Their number system was carried to the Western world by traveling Arabs. It became the system we use today. We use *Arabic numerals* (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) for expressing numbers.

1.08 The basis of our number system is the number ten. We call the system the *decimal system*. The word *decimal* comes from the Latin word *decem*, meaning ten.

**Fig. 1-1. Sets of screws**



**Fig. 1-2. Numeral divided into periods**



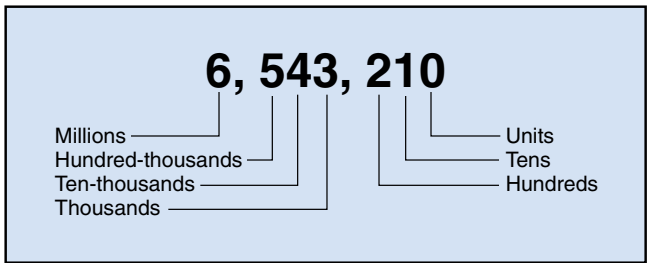
1.09 In the decimal system, digits are grouped in *periods* of three digits each. Each period after the first one has a name. Figure 1-2 shows this arrangement. Units are always in the right-hand position in a period. Tens are always in the middle, and hundreds are always in the left-hand position.

1.10 The periods are separated to simplify the reading of large numbers. In North America, commas usually separate the periods. In Europe, decimal points separate the periods. You may also see the periods separated by blank spaces.

1.11 There are only ten digits in the decimal system, including zero. Therefore, a number with a value greater than nine must be represented by two or more digits. For example, one more than nine is ten. The numeral for ten is written 10, which means one set of ten, plus zero units.

1.12 The largest numeral that can be written with two digits is 99. One more than ninety-nine is one hundred, written with three digits. Writing the numeral 100 means one set of hundreds, plus zero sets of ten, plus zero units.

**Fig. 1-3. Place values**



1.13 If a digit stands alone, it stands for a number of units. For example, the digit 2 stands for the number two. But suppose another digit—an 8, for example—is written to the right of the 2 to make the numeral 28. Then the digit 2 no longer stands for the number two. In this new position, it has a value of twenty, because the 2 is in the tens place. The numeral 28 stands for two sets of ten, plus eight units.

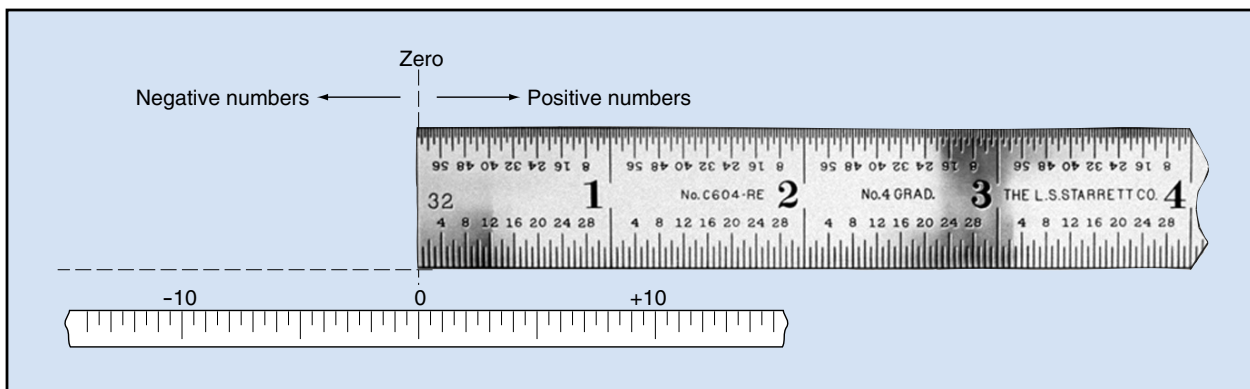
1.14 Suppose a third digit—for example, a 7—is added to the right of the 8. The value of the digit 2 then becomes two hundred, because the 2 is in the hundreds place. The numeral 287 stands for two hundreds, plus eight tens, plus seven units, or two hundred eight seven.

1.15 The position of each digit in a numeral shows its *place value*. Figure 1-3 shows the place values of the digits in a large numeral. That numeral stands for six million, five hundred forty three thousand, two hundred ten.

**Positive and Negative Numbers**

1.16 The numbers called *whole numbers* are the counting numbers (1, 2, 3, 4,...), plus the negatives of

**Fig. 1-4. Positive and negative numbers**



these numbers  $(-1, -2, -3, -4, \dots)$ , and zero. In other words, the set of whole numbers includes all numbers that are not fractions.

1.17 The digit 0 is very important in the decimal system. It indicates the presence of *nothing*. When measuring an object, for example the length of a board, zero is the starting point. On a ruler, zero is the point where there is no length. The starting point on the ruler in Fig. 1-4 is at the left end of the ruler.

1.18 On the thermometer scale below the ruler, zero is in the middle of the scale. Numbers greater than zero are called *positive*. Numbers less than zero are called *negative*. The negative numbers have a negative or *minus* sign  $(-)$  in front of them.

1.19 The negative sign is always used if a numeral stands for a negative number. The positive sign  $(+)$  may or may not be used. If a numeral has no sign, you can assume that it stands for a positive number. For example, the numerals 5 and  $+5$  both stand for positive five. The number zero is neither positive nor negative. It is the starting point from which the ideas of positive and negative numbers get their meanings.

### Addition

1.20 Addition is one of four basic operations of mathematics. *Addition* is the process of finding a number whose value equals the combined values of two or more given numbers. The plus sign  $(+)$  indicates the operation of addition.

$$\begin{array}{r} 3 \\ +4 \\ \hline 7 \end{array}$$

The numbers being added are called *addends*. Their combined value is called the *sum*.

1.21 Add the following numbers and write your answers in the spaces provided under each problem. The correct answers appear on page 20 of this lesson.

$\begin{array}{r} 8 \\ +2 \end{array}$	$\begin{array}{r} 9 \\ +2 \end{array}$	$\begin{array}{r} 5 \\ +7 \end{array}$	$\begin{array}{r} 8 \\ +5 \end{array}$	$\begin{array}{r} 9 \\ +1 \end{array}$	$\begin{array}{r} 6 \\ +6 \end{array}$	$\begin{array}{r} 8 \\ +3 \end{array}$	$\begin{array}{r} 5 \\ +8 \end{array}$
$\begin{array}{r} 6 \\ +5 \end{array}$	$\begin{array}{r} 7 \\ +6 \end{array}$	$\begin{array}{r} 7 \\ +8 \end{array}$	$\begin{array}{r} 3 \\ +4 \end{array}$	$\begin{array}{r} 4 \\ +3 \end{array}$	$\begin{array}{r} 8 \\ +8 \end{array}$	$\begin{array}{r} 4 \\ +3 \end{array}$	$\begin{array}{r} 5 \\ +3 \end{array}$

Fig. 1-5. Adding two-digit numbers

PROBLEM	THINK	WRITE
$\begin{array}{r} 15 \\ +23 \\ \hline \end{array}$	$\begin{array}{l} 1 \text{ ten} + 5 \text{ units} \\ +2 \text{ tens} + 3 \text{ units} \\ \hline 3 \text{ tens} + 8 \text{ units} \end{array}$	$\begin{array}{r} 15 \\ +23 \\ \hline 38 \end{array}$

1.22 When adding numbers that have two or more digits, the place value of each digit becomes very important. For example, suppose you must add 15 and 23. Begin this addition by writing the numerals so that the digits line up in columns, as shown in Fig. 1-5. The right column contains two sets of units  $(5 + 3 = 8)$ . The left column contains two sets of tens  $(1 \text{ ten} + 2 \text{ tens} = 3 \text{ tens})$ . To find the sum, you first add the two sets of units, and then add the two sets of ten. The sum is 38.

### Carrying

1.23 Suppose you are adding 56 and 28. You first add the 6 and 8. The sum is 14—four units, plus one ten. But, what do you do with that ten? You *carry* the ten from the units column to the tens column, as shown in Fig. 1-6.

1.24 In adding two-digit numerals, you first add the units. You write the number of units in the units column. Then, above the tens column, you write how many tens you are carrying. Include this number when you add the tens column.

Fig. 1-6. Carrying in addition  $(56 + 28 = ?)$

THINK	WRITE
$\begin{array}{l} 5 \text{ tens} + 6 \text{ units} \\ +2 \text{ tens} + 8 \text{ units} \\ \hline (7 \text{ tens} + \quad) + (1 \text{ ten} + 4 \text{ units}) \end{array}$ <p style="text-align: center;">Carry</p>	$\begin{array}{r} 1 \\ 56 \\ +28 \\ \hline 84 \end{array}$ <p style="text-align: right;">1 ten carried to this column</p> <p style="text-align: left;">4 units remaining in units column</p>

Fig. 1-7. Carrying in addition (27,239 + 92,337 + 1136 = ?)

THINK						WRITE
2 ten-thousands	+	7 thousands	+2 hundreds	+3 tens	+ 9 units	<sup>1</sup> 27,239
9 ten-thousands	+	2 thousands	+3 hundreds	+3 tens	+ 7 units	92,337
+		+ 1 thousands	+1 hundreds	+3 tens	+ 6 units	+ 1,136
( )+(1 hund. thou. + 1 ten thou. + )+(1 ten thou. + 0 thou.)+(6 hund. + )+(9 tens + )+(2 tens + 2 units)						120,712
Carry		Carry		Carry		
						(1 hund. + 1 ten)

1.25 Practice adding the following numbers. The correct answers appear on page 20 of this lesson.

$$\begin{array}{r} 64 \\ +27 \\ \hline \end{array} \quad \begin{array}{r} 77 \\ +16 \\ \hline \end{array} \quad \begin{array}{r} 49 \\ +22 \\ \hline \end{array} \quad \begin{array}{r} 64 \\ +23 \\ \hline \end{array} \quad \begin{array}{r} 17 \\ +53 \\ \hline \end{array} \quad \begin{array}{r} 28 \\ +67 \\ \hline \end{array}$$

1.26 You use the same procedure when carrying from the tens column to the hundreds column, from the hundreds column to the thousands column, and so on. Figure 1-7 shows a problem that requires carrying in several columns.

1.27 Add the following numbers. The correct answers appear on page 20 of this lesson.

$$\begin{array}{r} 2,287 \\ 9,954 \\ +1,386 \\ \hline \end{array} \quad \begin{array}{r} 33,042 \\ 128,600 \\ +65,936 \\ \hline \end{array} \quad \begin{array}{r} 317,675 \\ 1,240,443 \\ +533,502 \\ \hline \end{array} \quad \begin{array}{r} 768,265 \\ 1,838,921 \\ +662,973 \\ \hline \end{array}$$

**Shortcuts in Addition**

1.28 You can use two “tricks” to make addition problems easier to solve. One trick is to change the

order of the numbers. Changing the *order* does not change the answer. For example, suppose you want to add 2, 3, and 4. You can add them in any order, and the answer will always be 9.

$$\begin{array}{r} 2 \\ +3 \\ \hline 5 \end{array} \quad \begin{array}{r} 2 \\ +4 \\ \hline 6 \end{array} \quad \begin{array}{r} 3 \\ +2 \\ \hline 5 \end{array} \quad \begin{array}{r} 3 \\ +4 \\ \hline 7 \end{array} \quad \begin{array}{r} 4 \\ +2 \\ \hline 6 \end{array} \quad \begin{array}{r} 4 \\ +3 \\ \hline 7 \end{array}$$

1.29 The second trick is to change the way you *group*. Numbers can be regrouped in any way you want without changing the answer. For example, suppose you want to add the numbers 9, 4, 2, 1, and 6. You can regroup these numbers in your mind, as shown in Fig. 1-8. This regrouping makes the problem much easier to solve.

1.30 Figure 1-9 shows a shortcut that lets you add large numbers in your head. This problem is to add 47 + 36. Start with the first number, 47. Then add the 3 (meaning 3 tens) of the 36 to the 47. The result is 77. Next, add the 6 (meaning 6 units) of the 36 to the 77. The result is 83. You have added 47 + 36 in your head, without writing anything.

Fig. 1-8. Regrouping in addition

PROBLEM	THINK	WRITE
$\begin{array}{r} 9 \\ 4 \\ 1 \\ 2 \\ +6 \\ \hline \end{array}$		$\begin{array}{r} 9 \\ (4) \\ (1) \\ 2 \\ +6 \\ \hline 22 \end{array}$

Fig. 1-9. Shortcut to addition

PROBLEM	SAY TO YOURSELF	BECAUSE YOU...
47	47	(start)
+36	77	Add 3 tens to 47
Answer →	83	Add 6 to 77

**Fig. 1-10. Shortcut to addition**

PROBLEM	SAY TO YOURSELF	BECAUSE YOU . . .
257	257	(start)
368	557	Add 3 hundreds to 257
+109	617	Add 6 tens to 557
<hr/>	625	Add 8 units to 617
	725	Add 1 hundred to 625
Answer $\longrightarrow$	734	Add 9 units to 725

1.31 Figure 1-10 shows the same method used for adding three three-digit numbers. Remember to take each number in order. Always start with the left-hand digit, just as when you read the numbers.

1.32 Now practice addition by solving the problems below. Try adding them in your head. The correct answers appear on page 20 of this lesson.

$$\begin{array}{r}
 855 \\
 +56 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 216 \\
 +188 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 678 \\
 +298 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 189 \\
 +278 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 4278 \\
 +164 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 2165 \\
 +4894 \\
 \hline
 \end{array}$$

The Programmed Exercises on the following page will tell you how well you understand the material you have just read. Before starting the exercises, remove the REVEAL KEY from the back of the book. Read the instructions printed on the Reveal Key. Follow these instructions as you work through the Programmed Exercises.



## 10 Programmed Exercises

1-1. A number is a(n) _____. A numeral is a(n) _____.	1-1. IDEA; SYMBOL Ref: 1.01
1-2. The ten symbols for whole numbers from zero through nine are called _____.	1-2. DIGITS Ref: 1.05
1-3. Write the numeral that represents two sets of hundreds, plus eight sets of tens, plus seven units.	1-3. 287 Ref: 1.14
1-4. What does the digit 6 represent in the numeral 1688?	1-4. SIX HUNDREDS Ref: 1.15, Fig. 1-3
1-5. Numbers higher than zero are called _____ numbers, and those lower than zero are called _____.	1-5. POSITIVE; NEGATIVE Ref: 1.18
1-6. The process of finding the sum of two or more numbers is called _____.	1-6. ADDITION Ref: 1.20
1-7. When adding 78 and 26, one set of ten must be taken from the units column and added to the tens column. What is this operation called?	1-7. CARRYING Ref: 1.23
1-8. In addition, can you change the order of the numbers?	1-8. YES Ref: 1.28

## Multiplication

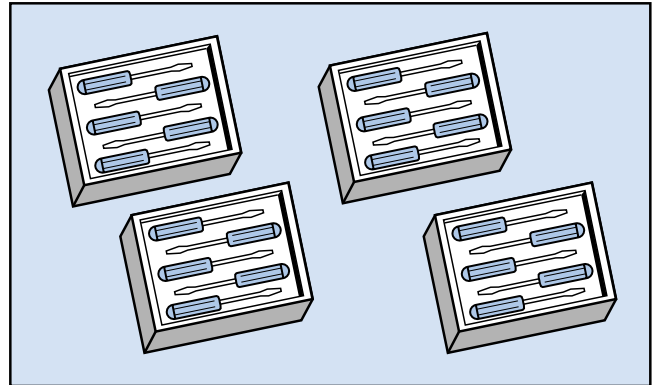
1.33 A second basic operation in mathematics is multiplication. Figure 1-11 shows four boxes, each containing five screwdrivers. What is the total number of screwdrivers in these boxes?

1.34 One way to find out would be to count them one-by-one. But this method is slow, especially if there are many boxes of screwdrivers. You could also solve the problem by adding  $5 + 5 + 5 + 5 = 20$ . But this method is also slow.

1.35 The best way to solve the problem is to multiply 4 (the number of boxes) by 5 (the number of screwdrivers in each box) to get the answer, 20. *Multiplication* is the repeated addition of identical numbers. Instead of counting each individual screwdriver, you count the screwdrivers in one *set*. Then you use multiplication to find the total number of screwdrivers.

1.36 The symbol for multiplication is an  $\times$ . The screwdriver problem would be written  $4 \times 5 = 20$ .

Fig. 1-11. Screwdriver sets



1.37 In order to solve multiplication problems, you must memorize all the *multiplication facts*. These facts are the products of all possible pairs of digits. For example,  $0 \times 1 = 0, 0 \times 2 = 0, \dots; 1 \times 1 = 1, 1 \times 2 = 2, \dots; 2 \times 1 = 2, 2 \times 2 = 4, \dots; 5 \times 3 = 15, 5 \times 4 = 20, \dots; 9 \times 8 = 72, 9 \times 9 = 81$ .

1.38 If you are unsure of the multiplication facts, you must work at memorizing them. You must know every possible combination if you are to multiply quickly and accurately.

Fig. 1-12. Multiplication

PROBLEM	THINK	WRITE
$\begin{array}{r} 37 \\ \times 25 \\ \hline \end{array}$	<p>First multiplication</p> $\begin{array}{r} 3 \text{ tens} + 7 \text{ units} \\ \times 5 \text{ units} \\ \hline 15 \text{ tens} + 35 \text{ units} \end{array}$ <p><math>(1 \text{ hundred}) + (5 \text{ tens} + ) + (3 \text{ tens}) + (5 \text{ units}) =</math></p> <p style="text-align: center;">Carry</p> <p>Second multiplication</p> $\begin{array}{r} 3 \text{ tens} + 7 \text{ units} \\ \times 2 \text{ tens} \\ \hline 6 \text{ ten} \times \text{ten} + 14 \text{ tens} \end{array}$ <p><math>(6 \text{ hundreds} + ) + (1 \text{ hundred}) + (4 \text{ tens}) =</math></p> <p style="text-align: center;">Carry</p> <p>Add partial products</p> $\begin{array}{r} 1 \text{ hundred} + 8 \text{ tens} + 5 \text{ units} \\ + 7 \text{ hundreds} + 4 \text{ tens} + 0 \text{ units} \\ \hline (8 \text{ hundreds} + ) + (1 \text{ hundred} + 2 \text{ tens}) + (5 \text{ units}) = \end{array}$ <p style="text-align: center;">Carry</p>	$\begin{array}{r} 3 \leftarrow \text{Carry} \\ 37 \\ \times 25 \\ \hline 185 \leftarrow \text{First partial product} \\ \\ 1 \leftarrow \text{Carry} \\ 3 \\ 37 \\ \times 25 \\ \hline 185 \\ \\ 37 \\ \times 25 \\ \hline 185 \\ + 74 \\ \hline 925 \leftarrow \text{Product} \end{array}$ <p style="text-align: right;">74 ← Second partial product</p>

**Fig. 1-13. Subtraction without borrowing**

PROBLEM	THINK	WRITE
$\begin{array}{r} 369 \\ -123 \\ \hline \end{array}$	3 hundreds + 6 tens + 9 units - 1 hundred - 2 tens - 3 units 2 hundreds + 4 tens + 6 units	$\begin{array}{r} 369 \\ -123 \\ \hline 246 \end{array}$

1.39 Test how well you know the multiplication facts by writing the products below. These are some of the hardest multiplication facts to remember. Check your answers on page 20 of this lesson.

$5 \times 6 = \underline{\quad}$   $6 \times 4 = \underline{\quad}$   $8 \times 8 = \underline{\quad}$   $9 \times 4 = \underline{\quad}$   
 $5 \times 8 = \underline{\quad}$   $7 \times 3 = \underline{\quad}$   $8 \times 9 = \underline{\quad}$   $9 \times 9 = \underline{\quad}$   
 $6 \times 7 = \underline{\quad}$   $7 \times 5 = \underline{\quad}$   $8 \times 7 = \underline{\quad}$   $9 \times 6 = \underline{\quad}$

1.40 When you multiply numbers, you must multiply every digit in one number by every digit in the other. Figure 1-12, on the previous page, shows how to do this for two two-digit numbers. You first multiply the top number (37) by the right-hand digit (5) in the bottom number. This multiplication gives you the *first partial product*.

1.41 Next, you multiply the top number (37) by the left-hand digit (2) in the bottom number (25). This multiplication gives you the *second partial product*.

1.42 If there were more digits in the bottom number, you would continue this same process. You would multiply the top number (37) by the next digit to the

left in the bottom number. This multiplication would give you a *third partial product* and so on.

1.43 Finally, after you have multiplied by all the digits in the bottom number, you add the partial products to get the *product*.

1.44 Practice multiplying the following numbers. The correct answers appear on page 20 of this lesson.

$$\begin{array}{r} 15 \\ \times 20 \\ \hline \end{array}$$

$$\begin{array}{r} 28 \\ \times 40 \\ \hline \end{array}$$

$$\begin{array}{r} 49 \\ \times 70 \\ \hline \end{array}$$

$$\begin{array}{r} 77 \\ \times 86 \\ \hline \end{array}$$

$$\begin{array}{r} 43 \\ \times 54 \\ \hline \end{array}$$

$$\begin{array}{r} 743 \\ \times 21 \\ \hline \end{array}$$

$$\begin{array}{r} 862 \\ \times 24 \\ \hline \end{array}$$

1.45 In multiplication, it makes no difference which number you write on the top and which you write on the bottom. Either way, the product is the same.

$$\begin{array}{r} 37 \\ \times 25 \\ \hline 185 \\ 74\phantom{0} \\ \hline 925 \end{array}$$

$$\begin{array}{r} 25 \\ \times 37 \\ \hline 175 \\ 75\phantom{0} \\ \hline 925 \end{array}$$

**Subtraction**

1.46 Suppose a box contains six screws. You remove four of them to install a mounting plate. Only two screws remain in the box. You took one set of four from another set of six. This operation is called *subtraction*. It is the opposite of addition. The subtraction sign (-) indicates the operation of subtraction.

1.47 You can subtract numbers in any order, just as you can add numbers in any order. But you must be

**Fig. 1-14. Subtraction with borrowing**

PROBLEM	THINK	WRITE
$\begin{array}{r} 455 \\ -239 \\ \hline \end{array}$	4 hundreds + 5 tens + 5 units - 2 hundreds - 3 tens - 9 units	$\begin{array}{r} 455 \\ -239 \\ \hline \end{array}$
	<p>Borrow</p> $\begin{array}{r} 4 \text{ hundreds} + (4 \text{ tens} + 1 \text{ ten}) + (\phantom{0} + 5 \text{ units}) \\ - 2 \text{ hundreds} - 3 \text{ tens} - 9 \text{ units} - 9 \text{ units} \\ \hline \end{array}$	<p>Amount remaining after borrowing</p> $\begin{array}{r} 455 \\ -239 \\ \hline 4\cancel{5}5 \\ -23\cancel{9} \\ \hline \end{array}$ <p>Amount borrowed</p>
	$\begin{array}{r} 4 \text{ hundreds} + 4 \text{ tens} + 15 \text{ units} \\ - 2 \text{ hundreds} - 3 \text{ tens} - 9 \text{ units} \\ \hline 2 \text{ hundreds} + 1 \text{ ten} + 6 \text{ units} \end{array}$	$\begin{array}{r} 455 \\ -239 \\ \hline 216 \end{array}$

sure to remember one important fact. *The subtraction sign stays with the numeral that follows it.* For example, suppose you have a problem involving both addition and subtraction.

$$\begin{array}{r} 6 - 4 + 3 = \\ \underbrace{6 - 4} + 3 = \\ 2 + 3 = 5 \end{array}$$

1.48 You can change the order of the numbers, but you must keep each plus sign and minus sign with its numeral.

$$\begin{array}{r} 6 + 3 - 4 = \\ \underbrace{6 + 3} - 4 = \\ 9 - 4 = 5 \end{array}$$

$$\begin{array}{r} 3 - 4 + 6 = \\ \underbrace{3 - 4} + 6 = \\ -1 + 6 = 5 \end{array}$$

$$\begin{array}{r} -4 + 6 + 3 = \\ \underbrace{-4 + 6} + 3 = \\ 2 + 3 = 5 \end{array}$$

1.49 Try solving these problems for practice. The correct answers appear on page 20 of this lesson.

$$\begin{array}{r} 2 \\ +3 \\ -1 \\ +4 \end{array} \quad \begin{array}{r} 9 \\ +4 \\ +2 \\ -6 \end{array} \quad \begin{array}{r} 2 \\ -7 \\ +5 \\ +4 \end{array} \quad \begin{array}{r} 3 \\ -4 \\ -5 \\ +9 \end{array}$$

1.50 To subtract numbers with two or more digits, arrange the digits in columns. Then, starting from the right, subtract the lower digit from the upper digit in each column. Figure 1-13 shows how to subtract one three-digit number from another.

### Borrowing

1.51 In many subtraction problems, you will need to subtract a larger digit from a smaller digit. For example, suppose you must solve this problem.

$$\begin{array}{r} 455 \\ -239 \\ \hline \end{array}$$

In the units column, you must subtract 9 from 5. The result is negative, which makes the subtraction confusing.

1.52 You can eliminate the confusion by borrowing. In *borrowing*, you shift value from one column to another in the upper numeral. If the digit in one column is too small in value, you add ten to that column. The ten comes from the next column to the left. The value in this column is then reduced by one.

1.53 Figure 1-14 shows how borrowing works. You borrow one from the tens column, and add ten to the units column. Doing so does not change the value of the upper numeral.

1.54 Borrowing in subtraction is the opposite of carrying in addition. You can borrow value from any column and add the same value to the next column to the right. Figure 1-15 shows another example of borrowing.

1.55 Practice subtracting by solving the following problems. The correct answers appear on page 20 of this lesson.

$$\begin{array}{r} 28 \\ -17 \end{array} \quad \begin{array}{r} 38 \\ -19 \end{array} \quad \begin{array}{r} 423 \\ -218 \end{array} \quad \begin{array}{r} 641 \\ -279 \end{array} \quad \begin{array}{r} 4110 \\ -3243 \end{array}$$

**Fig. 1-15. Subtraction with borrowing**

PROBLEM	THINK	WRITE
$\begin{array}{r} 752 \\ -457 \\ \hline \end{array}$	7 hundreds + 5 tens + 2 units -4 hundreds - 5 tens - 7 units	$\begin{array}{r} 752 \\ -457 \\ \hline \end{array}$
	$\begin{array}{r} \text{Borrow} \quad \quad \quad \text{Borrow} \\ \curvearrowright \quad \quad \quad \curvearrowright \\ (6 \text{ hundreds} + 1 \text{ hundred}) + (+4 \text{ tens} + 1 \text{ ten}) + (+2 \text{ units}) \\ -4 \text{ hundreds} \quad \quad \quad -5 \text{ tens} \quad \quad \quad -7 \text{ units} \\ \hline \end{array}$	$\begin{array}{r} 6 \cancel{7} \cancel{5} 2 \\ -4 \cancel{5} 7 \\ \hline \end{array}$
	$\begin{array}{r} 6 \text{ hundreds} + 14 \text{ tens} + 12 \text{ units} \\ -4 \text{ hundreds} - 5 \text{ tens} + 7 \text{ units} \\ \hline 2 \text{ hundreds} + 9 \text{ tens} + 5 \text{ units} \end{array}$	$\begin{array}{r} 6 \cancel{7} \cancel{5} 2 \\ -4 \cancel{5} 7 \\ \hline 2 \ 9 \ 5 \end{array}$

## Division

1.56 Suppose you want to distribute 12 rolls of tape equally among four mechanics. How many rolls of tape should each mechanic receive? To find out, you could hand one roll at a time to each mechanic in turn until you handed out all the rolls. But this method of dividing is slow, especially if you have 1200 rolls of tape and 400 mechanics. A better way is to figure out how many times 4 can be subtracted from 12.

1.57 *Division* is the repeated subtraction of one number from another number. It is the opposite of multiplication. You must know how to multiply before you can divide.

1.58 You can multiply and divide in any order. But you must remember to keep the operational signs with their numbers. The examples below show the same problem solved in different orders. Notice that the division sign stays with the 2, and the multiplication signs stay with the 4 and the 6.

$$\begin{aligned} \underbrace{4 \times 6} \div 2 &= \\ 24 \div 2 &= 12 \\ \underbrace{4 \div 2} \times 6 &= \\ 2 \times 6 &= 12 \\ \underbrace{6 \div 2} \times 4 &= \\ 3 \times 4 &= 12 \\ \underbrace{(1) \div 2} \times 4 \times 6 &= \\ \underbrace{1/2} \times 4 \times 6 &= \\ 2 \times 6 &= 12 \end{aligned}$$

1.59 You can often divide small numbers by other small numbers from memory. For example, you know that 12 divided by 3 equals 4, if you remember that 4 times 3 equals 12. Likewise, you know that  $56 \div 8 = 7$ , if you remember that  $7 \times 8 = 56$ .

1.60 In these examples, the numbers are all numbers that go together in your list of multiplication facts. But suppose one or both numbers in a division problem are not in your list of multiplication facts. For example, you probably cannot solve any of these problems from memory.

$$70 \div 3 = ? \quad 25 \div 4 = ? \quad 1290 \div 86 = ?$$

1.61 In order to solve these division problems, you need to follow a method that uses what you remember about the multiplication facts. This method is called *long division*. Figure 1-16 explains the steps in solving problems by long division. The problem being solved is  $1294 \div 15 = ?$

1.62 Practice long division by solving the following problems. If you have difficulty, study the steps shown in Fig. 1-16. The correct answers to these problems appear on page 20 of this lesson.

$$3311 \div 60 = \underline{\hspace{2cm}} \quad 429 \div 7 = \underline{\hspace{2cm}}$$

$$15,660 \div 33 = \underline{\hspace{2cm}} \quad 211,010 \div 120 = \underline{\hspace{2cm}}$$

Fig. 1-16. Long division

## STEPS

## THINK

## WRITE

1. Try dividing the divisor into the same number of digits in the dividend. In this case, try dividing 15 (a two-digit divisor) into 12 (first two digits of the dividend). It *cannot* be done in this problem, because the divisor is larger than the dividend (15 is larger than 12). If it *can* be done, skip to step 3.

$$\begin{array}{r} 15 \overline{)12\ 94} \\ \uparrow \qquad \uparrow \\ \text{Divisor} \quad \text{Dividend} \end{array}$$

$$15 \overline{)1294}$$

2. Add one more digit to the dividend.

$$15 \overline{)129\ 4}$$

$$15 \overline{)1294}$$

3. Guess at the quotient for this part of the problem. In this case, we will guess that the quotient is 7. Write the quotient above the last digit(9) in the dividend.

$$\begin{array}{r} 7 \\ 15 \overline{)129\ 4} \end{array}$$

$$\begin{array}{r} 7 \\ 15 \overline{)1294} \end{array}$$

4. Test your answer. Multiply the quotient by the divisor, and subtract the product from the dividend. Your quotient is correct if this difference is *not negative* and *less than the divisor*. In this case, the test shows seven is *not* the correct quotient, because the difference is larger than the divisor.

$$\begin{array}{r} 7 \\ 15 \overline{)129\ 4} \\ -105 \leftarrow 7 \times 15 \\ \hline 24 \leftarrow \text{Difference} \\ \qquad \qquad \qquad \text{(too large)} \end{array}$$

$$\begin{array}{r} 7 \\ 15 \overline{)1294} \\ \underline{105} \\ 24 \end{array}$$

5. Make another guess at the quotient. We will try nine. Test your answer.

$$\begin{array}{r} 9 \\ 15 \overline{)129\ 4} \\ -135 \leftarrow 9 \times 15 \\ \hline -6 \leftarrow \text{Difference} \\ \qquad \qquad \qquad \text{(negative)} \end{array}$$

$$\begin{array}{r} 9 \\ 15 \overline{)1294} \\ \underline{135} \\ -6 \end{array}$$

6. Make a third attempt, if necessary. Test your answer.

$$\begin{array}{r} 8 \\ 15 \overline{)129\ 4} \\ -120 \leftarrow 8 \times 15 \\ \hline 9 \leftarrow \text{Difference} \\ \qquad \qquad \qquad \text{(acceptable)} \end{array}$$

$$\begin{array}{r} 8 \\ 15 \overline{)1294} \\ \underline{120} \\ 9 \end{array}$$

7. When your test shows the value for the quotient is acceptable, "bring down" the next digit in the dividend.

$$\begin{array}{r} 8 \\ 15 \overline{)129\ 4} \\ -120 \downarrow \text{Bring} \\ \hline 9\ 4 \text{ down} \end{array}$$

$$\begin{array}{r} 8 \\ 15 \overline{)1294} \\ \underline{120} \\ 94 \end{array}$$

8. Find the second digit in the quotient by repeating steps 3 through 7. When you test the quotient, test this digit only.

$$\begin{array}{r} 8\ 6 \\ 15 \overline{)129\ 4} \\ -120 \\ \hline 9\ 4 \\ -9\ 0 \leftarrow 6 \times 15 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 86 \\ 15 \overline{)1294} \\ \underline{120} \\ 94 \\ \underline{90} \\ 4 \end{array}$$

**Fig. 1-16. Long division, continued**

**STEPS**

9. Continue this process until you have “brought down” and tested the last digit in the dividend. When you have found the last digit in the quotient, you are finished. Any amount of the dividend left over is called the *remainder*.

**THINK**

$$\begin{array}{r}
 86 \\
 15 \overline{) 1294} \\
 \underline{-120} \\
 94 \\
 \underline{-90} \\
 4 \text{ Remainder}
 \end{array}$$

**WRITE**

$$\begin{array}{r}
 86 \\
 15 \overline{) 1294} \\
 \underline{120} \\
 94 \\
 \underline{90} \\
 4
 \end{array}$$

10 Check your answer. Multiply the quotient by the divisor and then add the remainder. The result should equal the dividend. If it does not, there is an error somewhere in your work.

$$\begin{array}{r}
 86 \leftarrow \text{Quotient} \\
 \times 15 \leftarrow \text{Divisor} \\
 \hline
 430 \\
 86 \\
 1290 \\
 + 4 \leftarrow \text{Remainder} \\
 \hline
 1294 \leftarrow \text{Dividend}
 \end{array}$$

$$\begin{array}{r}
 86 \\
 \times 15 \\
 \hline
 430 \\
 86 \\
 1290 \\
 + 4 \\
 \hline
 1294
 \end{array}$$

PREVIEW  
COPY

PREVIEW  
COPY



## 18 Programmed Exercises

<p>1-9. Adding <math>8 + 8 + 8 + 8 + 8 + 8</math> can be simplified to _____ <math>\times</math> _____. The answer is _____.</p>	<p>1-9. <math>6 \times 8; 48</math> Ref: 1.35</p>
<p>1-10. In multiplication, does changing the order of the numerals change the answer?</p>	<p>1-10. NO Ref: 1.45</p>
<p>1-11. What is the symbol for subtraction?</p>	<p>1-11. SUBTRACTION SIGN (-) Ref: 1.46</p>
<p>1-12. Carrying in addition is the opposite of _____ in subtraction.</p>	<p>1-12. BORROWING Ref: 1.54</p>
<p>1-13. Solve the following problem: <math>416 - 188 =</math> _____.</p>	<p>1-13. 228 Ref: Figs. 1-14, 1-15</p>
<p>1-14. Division is the opposite of _____.</p>	<p>1-14. MULTIPLICATION Ref: 1.57</p>
<p>1-15. Does <math>16 \div 4</math> mean the same thing as <math>4 \div 16</math>?</p>	<p>1-15. NO Ref: 1.58</p>
<p>1-16. Solve the following problem: <math>2538 \div 18 =</math> _____.</p>	<p>1-16. 141 Ref: Fig. 1-16</p>

Answer the following questions by marking an "X" in the box next to the best answer.

- 1-1. The place value of a digit depends on its
- a. equality to other digits
  - b. numerals between zero and nine
  - c. position in a numeral
  - d. relation to 0
- 1-2. Which of the following numerals expresses the lowest value?
- a. 30
  - b. 0
  - c. -10
  - d. -30
- 1-3. The answer to an addition problem is called the
- a. addend
  - b. quotient
  - c. sum
  - d. unit
- 1-4. When you add  $14 + 19 + 28 + 37$ , how many sets of ten must be carried from the units column to the tens column?
- a. 0
  - b. 1
  - c. 2
  - d. 3
- 1-5. How many finished parts will a machine make in four hours if it makes six each hour?
- a. 4
  - b. 6
  - c. 10
  - d. 24
- 1-6. A production line normally turns out 400 parts per hour. Because of poor maintenance in the past, the line must be shut down for repairs every 3 hours of running time. The repairs take 1 hour. How much production is lost in each 24-hour (3 shifts) working day as a result of this downtime?
- a. 1200 parts
  - b. 2400 parts
  - c. 3600 parts
  - d. 7200 parts
- 1-7. In a problem involving both addition and subtraction, you can write the numerals in any order but you must not change the
- a. amounts carried
  - b. answer
  - c. order of the digits
  - d. plus and minus signs
- 1-8. The operating-time indicator on an engine reads 1940 hours. The engine requires servicing every 200 hours. It was last serviced when the indicator read 1820. How many operating hours remain until the next service is required?
- a. 80
  - b. 120
  - c. 200
  - d. 2020
- 1-9. A water tank holding 1450 gallons is leaking at the rate of 45 gallons per hour. About how long will it take for the tank to become empty?
- a. 3 hours
  - b. 19 hours
  - c. 32 hours
  - d. 100 hours
- 1-10. A finished casting requires 12 lb of metal. How many castings can be poured from 22,000 lb of metal?
- a. 54
  - b. 545
  - c. 1833
  - d. 2166

## SUMMARY

To solve math problems, you must first decide which of the basic operations to use—addition, multiplication, subtraction, or division. You might find that a problem requires using more than one operation.

This lesson explained whole numbers, numerals, and the place values of digits in numerals. It also explained addition, multiplication, subtraction, and division of whole numbers.

Make sure to use orderly work habits when you solve math problems. Follow step-by-step procedures that simplify your work and ensure the accuracy of your answers. Solving problems will become easier if you follow these steps:

1. Study the problem carefully, so that you understand it completely.
2. Determine what you need to figure out.
3. Separate what you know from what you need to know.
4. Decide which math operations you need to use in solving the problem.
5. Perform all the operations, and arrange your work neatly.
6. Label your answer with the proper units, based on the values of the units you started with. Take into account any changes you made as you solved the problem.
7. Check your answer to make sure it is correct.

## Answers to Problems

		<b>1.39</b>	30	24	64	36
			40	21	72	81
			42	35	56	54
<b>1.21</b>	10, 11, 12, 13, 10, 12, 11, 13, 13, 15, 16, 10, 11, 17, 12, 12	<b>1.44</b>	300; 1120; 3430; 6622; 2322; 15,603; 20,688			
<b>1.25</b>	91, 93, 71, 87, 70, 95	<b>1.49</b>	8, 9, 4, 3			
<b>1.27</b>	13,627; 227,578; 2,091,620; 3,270,159	<b>1.55</b>	11, 19, 205, 362, 867			
<b>1.32</b>	911, 404, 976, 467, 4442, 7059	<b>1.62</b>	55,R.11; 61,R.2; 474,R.18; 1758,R.50			

## Answers to Self-Check Quiz

- |      |    |                                  |       |    |                                 |
|------|----|----------------------------------|-------|----|---------------------------------|
| 1-1. | c. | Position in a numeral. Ref: 1.15 | 1-6.  | b. | 2400 parts. Ref: 1.40           |
| 1-2. | d. | -30. Ref: 1.18                   | 1-7.  | d. | Plus and minus signs. Ref: 1.48 |
| 1-3. | c. | Sum. Ref: 1.20                   | 1-8.  | a. | 80. Ref: 1.50                   |
| 1-4. | c. | 2. Ref: 1.24                     | 1-9.  | c. | 32 hours. Ref: Fig. 1-16        |
| 1-5. | d. | 24. Ref: 1.35-1.39               | 1-10. | c. | 1833. Ref: Fig. 1-16            |